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LETTER TO THE EDITOR

Critical properties near σ dimensions for long-range interactions

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Abstract. We show that the critical temperature for n-vector models with long-range interaction falling off at infinity as $1/r^{d+\sigma}$ vanishes when $d=\sigma$, provided n is larger than one. As a consequence, we calculate the critical exponents as power series in $(d-\sigma)$ up to second order.

In this letter we consider the critical properties of an n-vector model with long-range interaction, at a dimension at which the critical temperature becomes small. It is well known that long-range attractive interactions decaying in d dimensions as $1/r^{d+\sigma}$ $(0<\sigma<2)$ lead to critical properties which differ in many respects from those of short-range interactions. Fisher *et al* (1972) have shown that Landau theory holds for $d>2\sigma$ and for $d<2\sigma$ they have performed an expansion in powers of $2\sigma-d$. Ma (1973) has used the $1/n$ expansion for the same problem. In this letter we report the results of a renormalization group analysis of the low temperature expansion. The theory is analogous in many respects to the one appropriate to short-range forces near two dimensions (Brezin and Zinn-Justin 1976) and we shall use similar notations. It leads to a characterization of the critical behaviour in the vicinity of the dimension σ.

We shall first give the results of this analysis. The critical temperature vanishes when d approaches σ for any n larger than one, and is proportional to $(d-\sigma)/(n-1)$. At order $(d-\sigma)^2$ the critical exponents are

$$\eta=2-\sigma \tag{1}$$

$$\frac{1}{\nu}=d-\sigma+\frac{(d-\sigma)^2}{2(n-1)}\left[\psi(\sigma)-\psi(1)+\frac{2}{\sigma}+\pi\cot\left(\frac{\pi\sigma}{2}\right)\right]+\mathrm{O}(d-\sigma)^3 \tag{2}$$

in which the function ψ is the logarithmic derivative of the function Γ.

The exponent η remains, as in previous studies (Fisher *et al* 1972, Ma 1973), fixed to its classical value $2-\sigma$, to all orders in $(d-\sigma)$. For $n=1$ and $d>\sigma$, t_c goes to infinity in our model and, as for the similar case of the short-range XY model in two dimensions, it is presumably a feature of the continuous field theory that we have studied which would not be present in the lattice problem. If $n=1$ and $d=\sigma$ the function W vanishes identically and the theory is scale invariant for any value of the temperature. These results are obtained from the renormalization group equation fulfilled by the vertex functions:

$$\left[\mu\frac{\partial}{\partial\mu}+W(t)\frac{\partial}{\partial t}+\left(\frac{1}{2}\zeta(t)+\frac{W(t)}{t}-(d-\sigma)\right)H\frac{\partial}{\partial H}\right]\Gamma^{(N)}(p,t,H,\mu)=0 \quad (3)$$

in which t is the (renormalized) temperature and μ the arbitrary (inverse) length scale which defines the renormalized theory. In this problem $\zeta(t)$ and $W(t)$ are not independent and to all orders in t one can show that

$$\zeta(t)=d-\sigma-\frac{W(t)}{t}. \quad (4)$$

An explicit calculation gives (in the dimensional regularization method)

$$W(t)=(d-\sigma)t-(n-1)t^2-\frac{(n-1)}{2}t^3\left[\psi(\sigma)-\psi(1)+\frac{2}{\sigma}+\pi\cot\left(\frac{\pi\sigma}{2}\right)\right]+O(t^4). \quad (5)$$

We have included in the renormalized temperature the factor $2\pi^{d/2}/(2\pi)^d\Gamma(d/2)$. For n larger than one, one reads from equation (5) that the theory is asymptotically free in σ dimensions, and that there is an ultraviolet fixed point of order $(d-\sigma)/(n-1)$ for d larger than σ. Let us recall that the exponents are given by the equations:

$$W(t_c)=0, \qquad W'(t_c)<0 \quad (6a)$$

$$1/\nu=-W'(t_c) \quad (6b)$$

$$d-2+\eta=\zeta(t_c). \quad (6c)$$

Let us briefly sketch the underlying theory. We start with classical spin vectors of fixed unit length. Below T_c there are $(n-1)$ modes, transverse to the spontaneous magnetization. The long-distance limit of this theory leads to a continuous theory with an effective Hamiltonian which couples these $(n-1)\boldsymbol{\pi}$ modes. The partition function in a field H is thus given by the functional integral:

$$Z(H)=\int\prod_x\frac{\mathrm{d}\boldsymbol{\pi}(x)}{\surd(1-\boldsymbol{\pi}^2(x))}\exp\left(-\frac{1}{2T}\int\mathrm{d}^dX[\boldsymbol{\pi}\Delta^\sigma\boldsymbol{\pi}+\surd(1-\boldsymbol{\pi}^2)\Delta^\sigma\surd(1-\boldsymbol{\pi}^2)-2H\surd(1-\boldsymbol{\pi}^2)]\right)$$

in which the Laplacian raised to the power σ is defined in Fourier space.

Power counting indicates that this theory is renormalizable in σ dimensions. The structure of the renormalized theory is obtained through a temperature and a field strength renormalization, respectively called Z_1 and Z. However, the coefficient of p^σ in the inverse propagator of the $\boldsymbol{\pi}$ is not divergent and thus $Z_1=Z$.

The calculation of the divergent part of Z may be made by requiring that $\langle\surd(Z^{-1}-\pi^2)\rangle$ should be finite. This yields, up to two-loop order, in the dimensional regularization method

$$Z^{1/2}=1+\frac{(n-1)t}{2(d-\sigma)}+\frac{(n-1)(3n-5)}{8(d-\sigma)^2}t^2+\frac{(n-1)}{(d-\sigma)^2}t^2\left\{1+\frac{d-\sigma}{2}\left[\psi(\sigma)-\psi(1)+\frac{2}{\sigma}+\pi\cot\left(\frac{\pi\sigma}{2}\right)\right]\right\}+O(t^3),$$

from which $W(t)$ is calculated by the formula

$$W(t) = (d-\sigma)t\left(1 + t\frac{\mathrm{d}\ln Z}{\mathrm{d}t}\right)^{-1}$$

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